

Queen Mary, University of London
MAS317 Linear Algebra II, 2007
Coursework 4

This coursework should be handed in by 1.30 pm on MONDAY October 22nd in the orange box on the second floor in the mathematics building. Late coursework will not be accepted. Write your name and student number at the top and clip all the sheets together.

1. [20 marks]

Let A be an invertible $n \times n$ matrix over F . Prove the following results.

- (i) A can be obtained from I_n by a sequence of elementary row operations. (Use the Corollary to (2.18) in lectures. very short proof.)
- (ii) If B is an $n \times m$ matrix over F then the row rank of AB is equal to the row rank of B .
- (iii) If C is an $m \times n$ matrix over F then the row rank of CA is equal to the row rank of C .

2. [30 marks]

In this question you need not check that $\mathcal{B}$ and $\mathcal{C}$ are bases for $\mathbb{R}^4$.

Let $\mathcal{B}$ denote the ordered basis for $\mathbb{R}^3$ given by
 $\mathcal{B} = ((1, 0, 1), (1, 3, 0), (2, 0, 1))$.

- (i) Find the coordinate vector for (a, b, c) with respect to $\mathcal{B}$.
- (ii) Find the change of basis matrix from $\mathcal{C}$ to $\mathcal{B}$ where
 $\mathcal{C} = ((1, 1, 1), (1, 2, 0), (1, 0, 1))$.

3. [25 marks]

The bilinear form $f : \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$ is defined by

$$f((x_1, x_2, x_3), (y_1, y_2, y_3)) = 2x_1y_2 - 2x_2y_1 - x_2y_3 + x_3y_2.$$

- (i) Calculate $(f; E)$ where E is the standard basis for $\mathbb{R}^3$.
- (ii) Calculate $(f; \mathcal{B})$ where $\mathcal{B} = ((1, 0, 1), (1, 2, 0), (1, 1, 1))$.
- (iii) Is it possible to find a basis $\mathcal{C}$ for $\mathbb{R}^3$ such that $(f; \mathcal{C})$ has two rows of zeros? Explain why.
- (iv) Prove that $(f; \mathcal{C})$ has all its diagonal entries zero for every basis $\mathcal{C}$ for $\mathbb{R}^3$.

4. [25 marks]

Let $f : \mathbb{R}^3 \times \mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$ be a linear in all three places and have the following properties: $f(e_1, e_2, e_3) = 6$ and

if $u_1, u_2, u_3 \in \mathbb{R}^3$ then if two of these vectors are interchanged then the value of $f(u_1, u_2, u_3)$ is multiplied by (-1) .

[eg $f(u_2, u_1, u_3) = -f(u_1, u_2, u_3)$ etc.....]

- (i) Prove that if two of u_1, u_2, u_3 are equal then $f(u_1, u_2, u_3) = 0$.
- (ii) Prove that if $u_1 = au_2 + bu_3$ with $a, b \in \mathbb{R}$ then $f(u_1, u_2, u_3) = 0$.
and hence that $f((1, 1, 2), (2, 2, 0), (0, 0, 3)) = 0$.
- (iii) Calculate $f((1, 2, 2), (2, 2, 0), (0, 0, 3))$

[Hint: think first - don't forget (ii), and do notice how similar the three vectors are in the two cases and use this information.].